LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2012**

# MT 2811 - MEASURE THEORY AND INTEGRATION

Date : 06/11/2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

**ANSWER ALL QUESTIONS EACH QUESTION CARRIES 20 MARKS : 5 x 20 = 100**

1. (a) Define outer measure and show that it is translation invariant **(5)**

(OR)

(b) Prove that B is the algebra generated by each of the following classes: the open

intervals open sets, the sets, and the  sets. **(5)**

(c) Prove that the outer measure of an interval equals its length **(15)**

(OR)

(d) Prove that Not every measurable set is a Borel set. **(15)**

1. (a) State and prove Lebesque Monotone Convergence theorem. **(5)**

(OR)

(b) Prove that if *f* is a non negative measurable function then there exists a sequence **(5)**

of measurable monotonically increasing simple function such that.

(c) State and prove Fatou’s Lemma for measurable functions.  **(15)**

(OR)

(d) State and prove Lebesgue Dominated Convergence theorem. **(15)**

1. (a) Show that if  is a sequence in a ring ℜthen there is a sequence  of disjoint

sets of ℜ such that  for each i and  for each N so that

 **(5)**

(OR)

(b) Prove that with a usual notations the outer measure  on H(ℜ),and the outer measure outer measure defined by  on S( ℜ) and on  are the same. **(5)**

(c) Show that if  is a measure on a -ring  then the class of sets of the form 

for any sets E,N such that While N is contained in some set in of zero

measure is a -ring and the set function defined by is a

complete measure on . **(15)**

(OR)

(d) Prove that if is an outer measure on H(ℜ),. Let  denote the class of 

Measurable sets then Prove that is a - ring and restricted to is a complete

measure. **(15)**

1. (a) Prove that space is a vector space for . **(5)**



(OR)

(b) State and prove Minkowski’s inequality. **(5)**

(c) State and prove Jensen’s inequality. Also prove that every function convex on an open interval is continuous. **(15)**

(OR)

(d) Prove that  where  is convex on (*a*, *b*) and . Also prove that a differentiable function  is convex on (*a*, *b*) if and only if is a monotone increasing function. **(15)**

1. (a) Define the following terms: total variation, absolutely continuous, and mutually singular

with respect to signed measure. **(5)**

(OR)

(b) Let *v* be a signed measure on [X, S]. Construct the measures *v*+ and *v*- on [X, S] such that *v* = *v*+ - *v*- and *v*+ ┴ *v*-. **(5)**

(c) If ,, and are - finite signed measure on [X, S] and «,« then prove that . Also prove that a countable union of positive sets with respect to a signed measure *v* is a positive set. **(15)**



(OR)

(d) State and prove Hahn decomposition theorem. **(15)**